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Explanatory schema and the process of model building

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Abstract In this paper, we argue that rather than exclusively focusing on trying to determine if an idealized model fits a particular account of scientific explanation, philosophers of science should also work on directly analyzing various explanatory schemas that reveal the steps and justification involved in scientists' use of highly idealized models to formulate explanations. We develop our alternative methodology by analyzing historically important cases of idealized statistical modeling that use a three-step explanatory schema involving idealization, mathematical operation, and explanatory interpretation.

Keywords Explanation · Modeling · Idealization

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1 Introduction

The philosophy of science literature has recently focused on the explanations provided by highly idealized models (Ariew et al. 2015, 2017; Batterman and Rice 2014; Bokulich 2011, 2012; Godfrey-Smith 2006; Morrison 2015; Potochnik 2007, 2009; Rice 2015, 2017; Rohwer and Rice 2016; Strevens 2004, 2009; Weisberg 2007a, 2013).[1] In this paper, we will argue that philosophers of science should not be exclusively trying to determine if the explanation provided by a particular idealized model fits a general account of scientific explanation, or if it provides a distinctive kind of scientific explanation. Philosophers who adopt these approaches typically make normative claims about the modeling process by reverse engineering them from their preferred account of the explanatory product—i.e. their philosophical accounts of what it takes to be an (or a particular kind of) explanation are used to justify various normative claims about the modeling process used to formulate explanations. In contrast, we contend that philosophers of science ought to be directly investigating the steps and justification involved in various explanatory schemas used by scientists en route to formulating historically important exemplars of explanation. That is, our starting point will be the process of using an idealized model to formulate an explanation (i.e. the explanatory schema) rather than the product (i.e. the explanation itself).[2]

We will argue that two kinds of traditional approaches to investigating the explanations provided by models, exemplified by the accounts of Strevens (2009) and Lange (2012, 2013), have placed too strong an emphasis on the question of whether an explanation is causal or statistical. Although we will focus on these two accounts because of their influence, several other philosophers adopt the same basic methodologies of trying to justify idealized modeling by appealing to a general account of explanation, or by demarcating a particular kind of explanation and then using that account to justify the use of a particular model to explain (e.g. Batterman 2002; Craver 2006; Kitcher 1981; Lewis 1986; Pincock 2007, 2012; Reutlinger 2016; Salmon 1984; Woodward 2003). While this literature has provided many insights, we will argue that these methodologies mischaracterize many of the most important features of the modeling processes involved in the development of three historically significant model-based explanations: reversion explanation in biology, ideal gas theory, and population genetics.[3] That is, their answers to more traditional questions—e.g. what kind of explanation is provided by a particular idealized model—have led their views to suggest mistaken claims about the justification that ought to be given for various steps in the modeling process.

[1] In this paper, we follow several philosophers in characterizing abstraction as the "leaving out" of details and idealization as the intentional introduction of distortions or falsehoods into a model or theory (Weisberg 2007b). For a more detailed discussion of this relationship see (Rohwer and Rice 2016).

[2] We put things in terms of "formulating an explanation" because we want to be careful about equating the model itself with the explanation. Instead, we think of model-based explanations as explanations that necessarily appeal to a model in the explanation (Bokulich 2011, 2012). In addition, we want to investigate the use of idealized models to discover or reveal explanatory information before scientific modelers have a complete explanation on hand. In these cases, the model is used to discover and construct an explanation, but it may not *be* an explanation on its own.

[3] A notable exception to this lack of focus is Hughes' DDI account (Hughes 1997).

In contrast, we have no general account of explanation to appeal to, nor do we aim to give necessary and sufficient conditions for all instances of a kind of explanation. Rather, we will examine three important case studies from the history of science and extract an account of the justifications for various steps in the modeling process used to develop these exemplary explanations.[4]

We take for granted that the statistical laws appealed to in each of these historical cases are "always serviceable for explanation" (Galton 1877, p. 532).[5] Assuming that these models are serviceable for explanation is not a controversial assumption since the cases we discuss are widely accepted by both scientists and philosophers as providing satisfactory explanations.[6] Moreover, instead of trying to determine what *kind* of explanation these cases ultimately provide, our focus will be on answering three different questions about these cases:

(1) What are the steps used by scientific modelers who cite these models in their explanations?[7]
(2) What justifies each step of that modeling process?
(3) What conditions are involved in allowing for the modeling process to provide results that are serviceable for explanation in these (and other future) cases?

Our cases cover a range of scientific disciplines: Francis Galton's use of a statistical idealization to explain stability in biological inheritance, James Clerk Maxwell's construction of the ideal gas law to explain various regularities that hold between the macroscale properties of gases, and R. A. Fisher's use of idealized statistical models to explain various patterns of evolutionary change. Despite their varied physical systems, they all target similar *explananda* and employ a similar kind of explanatory strategy. In each case the phenomenon to be explained is a large-scale regularity that is true of the overall population, but is not necessarily true of any of the individuals that make up the ensemble (Ariew et al. 2015, 2017). In addition, all three cases purport to explain by appealing to the deductive consequences of a highly idealized statistical model. The key challenge of our paper is to extract a "general explanatory schema" that reveals, in each case, the steps and justification for employing an idealized statistical model to explain real-world ensemble-level regularities. The schema we identify involves three steps: idealization, mathematical operation, and explanatory interpretation.

Ultimately, our project is not to argue that these cases result in causal or statistical explanations, but instead to analyze more directly when and why scientists are justified in appealing to highly idealized statistical models in the process of formulating their

4 We do not have a more general account of justification to offer here—that is beyond the scope of this paper. We are simply using justification to mean having good reasons for performing various steps in the modeling process. Thus, we are focused on justifying various actions and inferences used in scientific modeling rather than justifying particular beliefs. Moreover, we are not discussing justifying the belief in a scientific explanation (i.e. confirmation) warranted by reasoning such as inference to the best explanation.

5 For additional details on the historical significance of the Galton case see Stephen Stigler (2010).

6 This isn't to say that these models might not also be used for other purposes (e.g. prediction), but these modelers are explicitly interested in providing explanations and the cases are widely held to be explanations by philosophers and scientists alike. If one wanted to argue that these cases are nonexplanatory, we think the onus is on philosophers to account for why they are so widely held to be exemplary explanations.

7 Michael Weisberg has several papers investigating this kind of question in his excellent discussion of what makes a scientist a modeler (Weisberg 2007b).

explanations. Extracting the key features of this explanatory schema and the justification behind each step of that schema, illustrates the fruitfulness of our contribution. We buck the dominant trend of attempting to identify necessary and sufficient conditions regarding the resulting explanatory product. Moreover, explicating the justifications for the different steps in the explanatory schema allows us to identify the conditions for when scientific modelers will be justified in appealing to highly idealized statistical models to formulate explanations in the future.

What is more, we think philosophical attempts to identify necessary and sufficient conditions for (kinds of) explanations will have minimal impact on the practice of science since scientific modelers are not particularly interested in whether their explanations are causal or statistical according to philosophical accounts of explanation. However, scientific modelers are, or at least should be, interested in *whether or not the processes and modeling decisions they use to generate those explanations are justified.* Unfortunately, philosophers of science have typically attempted to determine the justification for the steps used by scientific modelers by reverse engineering them from their preferred accounts of explanation. However, as we will argue, this reverse-engineering methodology leads to the mischaracterization of the justifications that ought to be provided for various steps in the modeling processes used to formulate explanations.

In the following section, we analyze three historical case studies and extracting a general explanatory schema that includes the justification of each step in that schema. Then, Sect. 3 lays out the generalized explanatory schema extracted from these cases and discusses several of its important features. Section 4 investigates the justification for applying an explanatory schema that can be accrued by successful use of the schema to develop explanations of similar explananda in the past. Next, in Sect. 5, we contrast our approach with others in literature by arguing that applying two traditional methodologies for investigating scientific explanation, exemplified by Strevens and Lange, lead to a mischaracterization of the explanatory structure and justifications involved in these historically important cases. The final section concludes and suggests that philosophers of science use our methodology to identify additional explanatory schemas in the future.

2 Three historical exemplars of scientific explanation

Our alternative methodology will focus on the three questions presented in the introduction: What are the steps used by scientific modelers who cite these models in their explanations? What justifies each step of that modeling process? What conditions are involved in allowing for the modeling process to provide results that are serviceable for explanation in these (and other future) cases? By identifying the parallels between three historically important cases, we will extract what we call a “general explanatory schema” for the development of these explanations that appeal to the deductive consequences of highly idealized statistical models to explain ensemble-level effects.

2.1 Case #1: Galton and reversion

As a first example, we will look at a historically important case of early statistical modeling from Galton (1877, 1892) (see Ariew et al. (2017) for a more detailed treatment of this case). Galton sought to explain a curious regularity of heredity. In general, parents with extreme qualities (e.g. extreme tallness) produce children with similar qualities. However, rarely are the children as extreme as their parents. Instead the children tend to revert back to the mean value of the general population. Galton's explanation of this "reversion to the mean" involves a key assumption: that the trait in question is normally distributed around a population mean with some degree of dispersion. Given this assumption, Galton could explain reversion as a deductive consequence of the features of the normal distribution: if the first generation is normally distributed, the second generation will show a normal distribution of about the same mean and dispersion. What is revolutionary about this case is that Galton was one of the first to provide an explanation of this kind of statistical regularity by appealing to deductive consequences of an idealized statistical model (Hacking 1990; Sober 1980; Ariew et al. 2017; Stigler 2010). The process Galton used to generate his explanation is as follows.

First, Galton made assumptions about there being a very large number of events that are statistically independent (Hacking 1990). By making these assumptions, Galton was able to represent the population using an idealized "curve of error"—i.e. a normal curve—as a proxy. We call this step "the idealization step" in the explanatory schema since it introduces an idealized model as a proxy for the real-world system(s). Galton's use of the idealized model as a proxy is justified because he was interested in a large-scale regularity that is largely independent of the details of the individuals in the population.[8] Indeed, Galton notes that the regularity is purely a mathematical phenomenon that applies across a diverse range of systems with different physical components. Galton understood that this kind of statistical regularity was largely independent of the properties of the individual components of the system: "The law of deviation is purely numerical; it does not regard the fact whether the objects treated of are pellets in an apparatus like this, or shots at a target, or games of chance, or any other of the numerous groups of occurrences to which it is or may be applied" (Galton 1877, p. 495). As a result, Galton suggests that we investigate a mathematical proxy that will admit of exact solutions: "let us suppose a typical case, in which the conformity shall be exact, and which shall admit of discussion as a mathematical problem, and find what the laws of heredity must then be to enable successive generations to maintain statistical identity" (Galton 1877, p. 493). Moreover, Galton had good reason to make assumptions that would allow him to use statistical modeling techniques because of the past success of these techniques when applied to similar large-scale phenomena by his

[8] It is important to note, however, that this statistical independence does not require that the individual-level events also be causally independent. Indeed, the events do not even have to be causal events. For this reason, instead of interpreting Galton as an instance of causal abstraction, or an attempt to isolate particular causal factors, we contend that what Galton is doing is using an idealized model as a proxy for the entire suite of causal processes involved in heredity.

contemporaries—such as Adolphe Quetelet (Stigler 2010).[9] As a result, Galton's use of the normal curve as an idealized representation of the population is justified because it enabled him to apply the statistical modeling techniques that had been successful in previous investigations of this kind of population-level phenomenon.

Next, Galton appealed to various mathematical operations in order to derive his explanandum as a consequence of the normal curve. From the assumption that the population is normally distributed, Galton was able to derive that in a second generation there will be (ideally) a normal curve of the about same mean and variance (Galton 1877, p. 513).[10] These mathematical derivations are ultimately justified by the legitimacy of the mathematical techniques and laws Galton used. Furthermore, Galton's performing this particular demonstration (or derivation) is justified by the fact that the result allowed him to show that the phenomenon of interest (i.e. reversion in the real-world population) approximates the behavior of the idealized normal curve reoccurring across generations (see Hacking 1990 and Stigler 2010 for more details). We will call this second step of Galton's modeling process the "mathematical operation" step.

Following the derivation of the stability of the normal curve across generations in the model population, Galton then needed to interpret how those results could be applied to explain the behavior of real-world populations that only approximate these behaviors. Galton's application of the derivations from the highly idealized model to explain the behavior of real-world populations is justified by the fact that Galton's idealized statistical model and the real-world system(s) he was investigating both meet certain minimal conditions—they involve numerous events that are statistically independent—and, therefore, will approximate the same patterns of large-scale behavior that are largely independent of the details of the individual-level events.[11] The connection between these large-scale behaviors and these minimal conditions is guaranteed by various statistical limit theorems; e.g. the central limit theorem and various laws of large numbers. Moreover, these conditions and the justification they provide for Galton's use of the normal curve to explain come in degrees. The more events in the actual population and the more statistically independent those events are, the more justified one is in appealing to the consequences of the idealized model for purposes of explanation.[12]

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[9] It is important to note, however, that while Galton and Quetelet were interested in similar target explananda, their explanations for that kind of explananda ended up being quite different because of how they interpreted the statistical results (Ariew et al. 2015).

[10] More specifically, "Galton did not strictly deduce it, but rather demonstrated it by the device of his shot-dropping machine, the quincrunx, in which an analogy of this effect could be observed. That led him to the remarkable thought: the phenomenon that puzzled him could be deduced from the fact (or assumption) that traits were distributed according to the standard statistical law, the law of errors" (Hacking 1990, p. 186). For more details of Galton's demonstrations see (Hacking 1990; Stigler 2010).

[11] This corresponds to what Batterman and Rice (2014) call having the model and the real-world system within the same *universality class*. In its most general form, universality is just a statement of the fact that drastically different physical systems will display similar patterns of large-scale behavior that are largely independent of the details of their physical components and interactions (Kadanoff 2013).

[12] Of course, several other conditions are also required in order to derive the central limit theorem; e.g. the random variables must all have finite variance. For the sake of simplicity, we only focus on the two

Galton recognized that certain statistical patterns—e.g. the stability of the normal curve across generations—are often approximated across a range of physical systems composed of different kinds of components and causal interactions. Systems that meet certain minimal conditions (to varying degrees) will approximate a normal curve and Galton was one of the first to exploit the stability of those conditions and patterns to apply idealized statistical models to explain large-scale phenomena (Hacking 1990). As Galton puts it, "The typical [statistical] laws are those which most nearly express what takes place in nature generally; they may never be exactly correct in any one case, but at the same time they will always be *approximately true and always serviceable for explanation*" (Galton 1877, p. 532, emphasis added). We think this illustrates that Galton's goal for developing these statistical models is to provide an explanation of the observed patterns. Moreover, Galton's case illustrates how we can justify the application of results obtained from highly idealized statistical models to real-world systems by showing that the model and the real-world systems both meet the minimal conditions required to display (or approximate) certain stable large-scale patterns of behavior. As a result of meeting these conditions to varying degrees, real or possible system(s) of interest to the scientific modeler will approximate the same large-scale patterns of behavior as the idealized model they are using as a proxy. We will call this final step the "explanatory interpretation" step. In sum, the overall structure of Galton's use of a statistical model to generate an explanation involves the three steps of an explanatory schema: idealization, mathematical operation, and explanatory interpretation.

2.2 Case #2: statistical modeling in physics

As a second example, we will consider the ideal gas law, which states that $PV = \mathrm{nR}T$ where P is pressure, V is volume, T is temperature, n is the number of moles of gas, and R is the constant. As before, the explananda in this case are large-scale regularities between statistical properties of the ensemble that are largely independent of the properties of the individual components of the system. More specifically, the ideal gas law can be used to explain various macroscale regularities such as why, across a wide range of different types of gases, when the temperature of a gas enclosed in a fixed volume increases from T to T*, its pressure will increase from P to P*.[13] Moreover, these ensemble-level regularities are found to hold across numerous gases that are radically different in the types of molecules and interactions occurring in the gas. Consequently, the explananda are statistical pattern(s) at the ensemble level that are not dependent on (at least most of) the features of the individual molecules in the system(s).

Footnote 12 continued
conditions discussed above since we think the justification for these other assumptions will parallel the justification discussed above.

13 This is just one example of the kind of change that could be explained by appealing to the ideal gas law. However, what scientists really want to explain with the ideal gas law are the ensemble-level regularities across many different changes in the pressure and temperature of gases.

Like Galton's use of the normal curve to investigate reversion, the explanations for these ensemble-level regularities are developed by appealing to the consequences of an idealized statistical model.[14] The mathematical frameworks used in deriving the statistical model equation $PV = \mathrm{n}\mathrm{R}T$ require the following idealizing assumptions:

(1) The gas consists of a large number of identical molecules in constant random motion.
(2) The motions of each of the particles are statistically independent of one another.
(3) The volume occupied by the gas molecules is infinitesimally small compared to the volume of the container; i.e. the molecules do not take up any space.
(4) The molecules exert no long-range forces on one another and there are no inter-molecular forces between the molecules.
(5) Collisions between the molecules and the walls of the container are perfectly elastic (or are simply assumed not to occur).
(6) The gas obeys the processes of classical Newtonian mechanics (Moore 2003, p. 23).

These assumptions drastically distort the components and interactions of real gases in order to allow for the application of mathematical modeling techniques used to discern mathematical relationships between large-scale properties of gases. Without these assumptions, the mathematical techniques that allow for the derivation of the system-level equation $PV = \mathrm{n}\mathrm{R}T$ would not be applicable.

For example, the ideal gas law uses the Maxwell-Boltzmann distribution for the velocities of molecules. This statistical distribution is derived by imposing a particular probability distribution on the microstates of the system and then averaging over those microstates to discern macroscale properties of gases. More specifically, the Maxwell-Boltzmann distribution requires that one assume that the molecules are in constant random motion and the velocity components of each of the molecules (and across molecules) are statistically independent of one another. Similar to Galton's use of a statistical model, making these idealizing assumptions enables one to model the speed of the molecules using a Gaussian distribution (i.e. a normal curve) as a proxy for the actual speeds of the molecules. Importantly, however, the Maxwell-Boltzmann distribution applies only to ideal gases. In real gases, there are several additional factors (e.g. van der Waals interactions, vertical flows, relativistic speed limits, and quantum exchanges) that make the gas particles' speeds often very different from those specified by the Maxwell-Boltzman distribution. Despite these distortions, making these assumptions allows physicists to use the ideal gas law as a proxy for real-world gases to investigate various large-scale statistical patterns that are not dependent on the details of the individual particles.

The introduction of these idealizing assumptions in order to apply statistical modeling techniques constitutes the "idealization step" of our explanatory schema. This step is, again, justified because the phenomenon of interest is a large-scale regularity where the explanandum is not dependent on various features of the individual particles in the system. As a result, scientific modelers can justifiably use idealizing

[14] This equation is a statistical model because pressure and temperature are ensemble-level averages that are defined within the framework of statistical mechanics.

assumptions that distort the physical details of the components of the system in order to apply mathematical frameworks that allow them to focus on stable patterns at the ensemble level. Consequently, the use of these statistical models as a proxy for the real-world system is similar to Galton's use of statistical modeling. In both cases, the nature of the explanandum justifies making certain idealizing assumptions in order to apply the mathematical modeling techniques that have been previously successful for investigating that kind of explanandum.

Once scientific modelers have constructed an idealized statistical model to serve as a proxy, they can use various mathematical modeling techniques to derive the relationships between macroscale properties of the ideal gas. In the gas law case, after constructing the statistical model as a proxy, physicists can use various laws of statistics (and axioms of probability theory) to derive that $PV = \mathrm{nR}T$, where pressure and temperature are defined in terms of the statistical framework being applied within the model (Moore 2003; Morrison 2015; Woody 2015). This model can then be used to derive how various changes in the pressure, temperature, or volume of the ideal gas will result in changes in the other macroscale properties of the gas. Moreover, since this model equation refers only to the ensemble level properties of the system, it can be used to explain large-scale regularities concerning changes in pressure, temperature, and volume that range over many heterogeneous gases. These derivations of regularities are ultimately justified by the legitimacy of the mathematical modeling techniques used in generating the ideal gas equation and deriving various results from it. Moreover, the statistical modelers are justified in performing these specific derivations because they show how their explanandum approximates a deductive consequence of the idealized model they are using as a proxy.

After deriving various consequences from the ideal gas equation that relates pressure, volume, and temperature, the modeler then needs to interpret how those mathematical derivations can be applied to explain real gas behavior. As before, the application of the consequences of the highly idealized model to explain the behavior of real-world system(s) is justified by the fact that both the model and the real-world system(s) meet certain minimal conditions required to approximate the large-scale regularities captured by the ideal gas law. In this case, as before, the key minimal conditions are that there are *numerous* particles in constant random motion, whose velocities are largely *statistically independent*. As a result, we can again apply various statistical limit theorems (e.g. the central limit theorem) to show why various (real, possible, and model) gases that meet these minimal conditions, to various degrees, will behave in ways that approximate the behavior of an ideal gas.[15] Because both the ideal system and the real-world system(s) meet these minimal conditions to some degree, they will display similar large-scale patterns of behavior; i.e. they will display similar statistical regularities despite perhaps drastic differences in the features of their individual components. As before, this use of a highly idealized statistical model to develop an explanation instantiates the three steps of a general explanatory schema: idealization, mathematical operation, and explanatory interpretation.

[15] It is also worth noting that, as was the case with Galton's use of statistical laws, this application is not an instance of causal abstraction—i.e. abstractly describing an isolated causal process. Instead, the distribution is a proxy for the total suite of underlying events that need not be causal events.

2.3 Case #3: statistical modeling in evolutionary biology

Finally, parallel applications of statistical modeling can be found in population biology (Ariew et al. 2015; Walsh et al. 2002). Indeed, the foundational assumptions made by R. A. Fisher in developing population genetics were inspired by the assumptions underlying the kinetic theory of gases (Morrison 1996, 2004, 2015). In fact, Fisher himself explicitly tells us that: "The whole investigation may be compared to the analytical treatment of the Theory of Gases" (Fisher 1922, p. 321). Moreover, Fisher's evolutionary models have been touted as paradigmatic cases of explanation in the literature (Morrison 2015; Rice 2015; Sober 1983). Like Galton and Maxwell and Boltzmann, Fisher's general approach was to make various idealizing assumptions about the nature of the individual components of the system and their interactions in order to develop idealized statistical models to serve as proxies with which to investigate the large-scale behaviors of populations. The success of Fisher's approach was due to his *replacing* actual populations with highly idealized model populations that relied on the statistical modeling frameworks he adopted from gas theory. Indeed, Fisher's work on population genetics was able to develop various explanations "only by invoking a very *unrealistic*…model of a population" (Morrison 2015, p. 24).[16] As Margaret Morrison explains:

> … random mating as well as the independence of the different factors were also assumed. Finally, and perhaps most importantly, he assumed that the factors were sufficiently numerous so that some small quantities could be neglected; in other words, large numbers of genes were treated in a way similar to large numbers of molecules and atoms in statistical mechanics. As a result, Fisher was able to calculate statistical averages that applied to populations of genes in a way analogous to calculating the behaviour of molecules that constitute a gas. (Morrison 2004, p. 1197)

In other words, Fisher assumed that a population of genes could be treated like a population of gas molecules. Assuming that the genes in the populations are numerous and that mating is random and statistically independent allowed Fisher to apply the central limit theorem, which tells us that such a sample will approximate the normal distribution. Then, by assuming the population of genes is infinitely large, one can apply various laws of large numbers to eliminate sampling error (in this case genetic drift).[17] Finally, Fisher averaged over the individual events in order to identify statistical features of the overall distribution of genotypic traits and their fitnesses. In combination, these assumptions allowed Fisher to assume biological populations could be modeled as a normal distribution of trait fitnesses whose mean value was the expected outcome (presumably due to natural selection).

[16] We should also note that while Morrison calls Fisher's model abstract, she uses mathematical abstraction to mean what other authors have called ineliminable idealizations (Batterman 2002; Batterman and Rice 2014; Rice 2012, 2015). For more details, see (Morrison 2015, p. 21). Indeed, like the other two cases, there is no causal abstraction going on here. Instead, Fisher is using his idealized model population as a proxy for evolving systems.

[17] The law of large numbers states that as the samples size increases, the average of the quantities sampled will be increasingly closer to the expected outcome of those quantities.

With Fisher's statistical modeling we again see that when certain idealizing assumptions are made about the components of the system, scientists can use various statistical modeling techniques to construct models with which to investigate large-scale regularities approximated by real-world systems. For example, Fisher's statistical modeling framework can be used to derive that the sex ratio for a wide range of populations should be roughly 1:1 (Fisher 1930; Sober 1983). In asking why the sex ratio is often approximately 1:1, Fisher sought to explain an ensemble-level statistical regularity across many heterogeneous populations. Targeting this kind of explananda justified him in making several idealizing assumptions that enabled him to use the statistical modeling techniques that had been successfully used to explain similar kinds of explananda in gas theory. In addition to the random mating and statistical independence assumptions mentioned above, Fisher's sex ratio model also assumed that the population is infinite in size, that organisms reproduce asexually, each has equal access to resources that can be invested in raising offspring, and that the sex of the offspring is under the direct genetic control of the parent.

As before, Fisher's use of an idealized statistical model as a proxy—i.e. what we have been calling the "idealization step"—is justified because his explanandum is a large-scale regularity where what it is true of the ensemble-level patterns is largely independent of the various features of the individual organisms in the population. Indeed, the equilibrium sex ratio depends only on ensemble-level tradeoffs between average costs of offspring for the whole population. In addition, these various idealizing assumptions are necessary for applying the mathematical modeling frameworks Fisher used in developing explanations for these kinds of large-scale statistical regularities.

Once these idealized statistical models have been constructed as a proxy for the real-world system(s), as in the previous two cases, Fisher was able to use various laws that govern statistical distributions to derive relationships between large-scale properties of the population, e.g. its substitution cost and equilibrium sex ratio. The substitution cost tells us how the average resources required to raise a son (C_1) are related to the average resources required to raise a daughter (C_2). The ratio of these averages (C_2/C_1) is the substitution cost for the population and can be used to derive a statistical equation for the equilibrium sex ratio of the population, r, where $r = C_2/(C_1 + C_2)$ (Charnov 1982, p. 28). In Fisher's original model, the cost of male and female offspring is assumed to be the same, so we get a resulting equilibrium sex ratio of 1:1. The reason this is the equilibrium ratio is that, if males and females cost the same amount of resources, then parents who produce the minority sex would receive a higher reproductive value for the same amount of resources since their offspring would have a higher chance of being represented in subsequent generations than offspring of the majority sex. As Fisher puts it, since "each sex must supply half the ancestry of all future generations of the species…those parents, the innate tendencies of which caused them to produce [the minority sex]…would be progenitors of a larger fraction of future generations than would parents having a congenital bias towards the production of [the majority sex]. Selection would thus raise the sex ratio until the expenditure on the majority sex became equal to the expenditure on the minority sex" (Fisher 1930, pp. 142–43). These derivations are justified by the legitimacy of various statistical laws and other mathematical modeling techniques. Moreover, Fisher was justified in performing those particular derivations because they demonstrate that the explanandum approximates a consequence of the idealized statistical model he was using as a proxy. These derivations constitute the "mathematical operation" step in which the phenomenon is shown to approximate a consequence derived from the idealized statistical model.

Following the derivation of various statistical results about the distribution of traits in the model population, Fisher needed to determine how those results could be applied to explain the patterns observed in real-world populations. This is the final "explanatory interpretation" step. As in the above two cases, the use of idealized statistical models to explain ensemble-level regularities can be justified by determining that the highly idealized model system(s) and the real-world biological population(s) both meet certain minimal conditions required to approximate various large-scale statistical patterns of behavior. As a result of meeting these minimal conditions, to varying degrees, the model and the real-world population(s) will display (or approximate) the same large-scale patterns of behavior, despite differences in the details of their components and interactions. As before, what connects these minimal conditions with the large-scale behavior of interest are various limit theorems (e.g. the central limit theorem), which demonstrate that the more events there are, and the more statistically independent those events are, the more closely the real-world distribution will approximate the behaviors of the idealized statistical population.[18] Therefore, as in the previous two cases, we once again see the three steps of the general explanatory schema: idealization, mathematical operation, and explanatory interpretation.

3 A new methodology: extracting explanatory schemas

Our alternatively methodology has sought to directly investigate what justifies each of the steps used in appealing to an idealized model in order to develop an explanation. In each of the three cases above, the answer, we argue, is given by identifying the justifications involved in a general explanatory schema consisting of three steps:

(1) Idealization: the use of an idealized model as a proxy for the real-world system(s) in order to investigate large-scale patterns via various mathematical frameworks; e.g. using statistical modeling techniques in order to investigate regularities that involve ensemble-level properties of the population distribution.[19]
(2) Mathematical operation: discovering various consequences of the mathematical model; e.g. by applying statistical laws to demonstrate that the explanandum approximates a derivable consequence of the idealized model.
(3) Explanatory interpretation: the application of the mathematical results to the real-world phenomenon for purposes of explanation; e.g. showing that the explanandum is an instance of the large-scale statistical pattern that is approx-

[18] Furthermore, as additional applications of the model are found, more and more justification for employing those idealized models to develop explanations of similar statistical phenomena is accumulated

[19] We only claim that this is the role of the idealization step within our particular explanatory schema. It is not a general definition of idealization. Indeed, idealizations are used in a variety of ways (Rohwer and Rice 2013; Weisberg 2007a).

imated by various systems that meet certain minimal conditions (to varying degrees).[20]

There are several additional things to note about this explanatory schema. First, the steps of this schema are not intended to be necessary and sufficient conditions for providing (or formulating) a particular kind of explanation. This is not intended to be a general account of all statistical explanations. Rather, understanding this general schema identifies some important parallels between these historically important cases and spells out the justification for the steps in the modeling process used to develop these explanations. Second, the term ‘steps’ might misleading imply that our schema requires a temporal order of operations that must be followed by scientific modelers. While this temporal ordering is somewhat artificial, separating these steps allows us to distinguish various contexts of the modeling process in which different questions concerning justification arise. Of course, in many actual cases, these contexts will overlap and may arise in alternative orders than what is presented above.[21] Third, while we illustrate our new methodology by focusing on cases that involve the use of statistical modeling, we want to emphasize that this methodology can be used to explicate other explanatory schemas extracted from alternative sets of cases. For example, Batterman and Rice (2014) could plausibly be interpreted as using a similar methodology to extract various steps in a different explanatory schema they call providing a “minimal model explanation”.[22] In addition, we think our alternative methodology could be easily applied to identify the steps and justifications involved in developing explanations via various kinds of causal or mechanistic modeling. For example, by looking at historically successful cases of mechanistic modeling, we should be able to extract a set of modeling processes and conditions that are sufficient to justify the appeal to mechanistic models in order to formulate an explanation.[23]

It is also important to notice that the nature of the justification for each of these steps in the schema depends on the context in which the question about justification arises. In the context of the idealization step, the question is: what justifies the use of a particular kind of idealized model as a proxy when attempting to develop an explanation of a phenomenon? In the three cases analyzed above, using the idealized statistical model as a proxy is justified because it enables scientific modelers to investigate large-scale regularities with the best modeling techniques for developing an explanation of that kind of explanandum. In other words, the justification, in the

[20] Of course, showing that the model is predictively accurate in the way described in step (2) might be an essential part of providing an explanatory interpretation of the model in step (3). Despite this connection, we think it is useful to separate the predictive accuracy of the model from the identification of the minimal conditions on which the explanandum counterfactually depends. Thanks to an anonymous reviewer for helping us clarify this point.

[21] Indeed, in our first historical case from Galton it is difficult to distinguish the three steps from one another. This is not particularly surprising, however, because Galton was developing novel techniques that would later be analyzed in more detail and applied in a wide range of cases.

[22] The steps involved in developing these explanations are: (1) demonstration that most of the features of a class of systems are irrelevant to their large-scale patterns of behavior, (2) using that information to delimit a universality class of systems that will display that behavior, and (3) showing that the idealized model and the real-world target system(s) are both within that universality class.

[23] Craver’s (2006, 2007) work can be seen as engaging in this project for mechanistic modeling.

context of investigation, is provided by the nature of the explanandum in combination with the fact that certain modeling frameworks have proven to be particularly fruitful ways of developing explanations of that kind of explanandum. Since the above scientific modelers' explananda are precisely the kind of phenomena that statistical modeling techniques are well suited for, in the context of investigation, these modelers are justified in making various idealizing assumptions that enable them to construct an idealized statistical model to use as a proxy for the real-world system(s).

The second step in the explanatory schema is the use of various mathematical techniques to derive certain results. In this context, the key question concerning justification is: what justifies the derivation of particular consequences of the idealized model? In the three cases analyzed here, by applying statistical laws, probability axioms, and other mathematical modeling techniques, scientists can demonstrate that the explanandum approximates a mathematical consequence of their statistical models. Part of the justification for these mathematical demonstrations is provided solely by the legitimacy of the mathematical modeling techniques being employed.[24] In addition, however, the particular derivations performed by the modeler are further justified by the fact that they are necessary to demonstrate that the model exhibits the particular large-scale regularity the modeler is interested in explaining. As R. I. G. Hughes puts it, mathematical models "contain resources which enable us to demonstrate the results we are interested in" (Hughes 1997, p. S332). In sum, the second step is justified when the mathematical techniques are legitimate and are necessary for demonstrating that the explanandum approximates a consequence of the idealized model.

After demonstrating that the explanandum approximates a consequence of the idealized model, the modeler must interpret how those results can be used to provide an explanation of the phenomenon that occurs in the real-world system(s). In the context of this explanatory interpretation step, the key question concerning justification is: what justifies the modeler in appealing to the derived consequences of the idealized model for purposes of explaining real-world phenomena? In the cases analyzed here, the application of the results of the highly idealized model to the real-world system(s) is justified by the fact that both the model and the real-world system(s) meet certain minimal conditions that guarantee that they will approximate certain large-scale statistical patterns of behavior. In these cases, the occurrence of these statistical patterns requires only that very minimal conditions hold in the real-world population under investigation—e.g. that the population is sufficiently large and events are (for the most part) statistically independent of each other—in order for the application of the idealized model to a particular system to be justified. In the statistical modeling cases above, the link between these minimal conditions and the large-scale statistical behaviors is derivable from various statistical limit theorems, but this kind of link can be established in other ways as well.

For example, Batterman and Rice (2014) discuss "minimal model explanations" that focus on demonstrating that most of the systems' physical details are irrelevant in order to delimit a universality class of systems that will display similar behaviors at macroscales. Then, by showing that an idealized model is within the same uni-

[24] Indeed, many of these derivations are mathematically necessary. Therefore, to deny them would be irrational. Thanks to an anonymous reviewer for helping us see this point more clearly.

versality class as various real-world systems, Batterman and Rice argue that we can justify the use of the so-called "minimal model" to develop explanations of the behaviors displayed by systems in that class. Moreover, as a byproduct of delimiting the universality class, we can also identify a set of minimal conditions (e.g. locality, conservation, and symmetry) that are necessary for the phenomenon to occur (Batterman and Rice 2014, p. 360). Consequently, similar to our use of minimal conditions and limit theorems, appealing to universality classes uses certain minimal conditions and various mathematical modeling techniques (e.g. renormalization) to establish a more precise connection between a model and a real-world system that is able to justify the use of that minimal model in the development of an explanation. Establishing this link between the minimal conditions and the pattern of interest is sufficient to justify the use of an idealized model to explain because the model can then be used to identify key features of the system that the explanandum counterfactually depends on (Bokulich 2011, 2012; Rice 2015; Woodward 2003).[25]

Despite the crucial role of minimal conditions, in many cases, scientific modelers do not explicitly demonstrate that the model and the real-world system(s) necessarily meet the same minimal conditions. Instead, scientific modelers often seem to be using something like an inference to the best explanation in the following sense: the best explanation for why the model and the real-world system behave in the same way is that the pattern of interest is stable across systems with different physical components and interactions. In other words, scientists use the fact that the consequences of the idealized model are approximated by the behaviors of the real-world system(s) as evidence that the model and real-world system(s) are members of a class of systems that will display similar patterns of large-scale behavior despite (perhaps drastic) differences in their physical details. In many cases, this may be sufficient to justify the final step of the explanatory schema when explicitly demonstrating that the minimal conditions are met is difficult (or perhaps impossible).

While this connection between the idealized model and the real-world system depends on notoriously vague notions like approximation or similarity, the crucial role of limit theorems and their connection with minimal conditions provides a more mathematically precise link between the behavior of the model and the real-world system(s). Specifically, because the number of events and the independence of those events are directly tied to how well the system will approximate the behaviors involved in these limit theorems, we can use those minimal conditions to get a handle on how well the model's behavior will be approximated by the real-world system(s) of interest. For example, in biological modeling, the larger the real-world population, the more closely the system will approximate the behaviors of statistical models that depend on the condition of a sufficiently (or perhaps infinitely) large population. Similarly, establishing universality classes in the way mentioned above can help elucidate the approximation relation by making the connection between models and real-world systems more mathematically precise (and derivable). Although these kinds of derivations

[25] It is important to note that, while a counterfactual dependence will sometimes hold in virtue of a causal dependence, it does not require a causal dependence; e.g. a phenomenon could counterfactually depend on a statistical feature of the population such as is mean, variance, or population size.

will not apply in every case, they can provide a more precise sense of what is meant by approximation in the cases analyzed here.

In addition, while meeting these conditions "to some degree" is admittedly vague, we think it captures some import features of how justification works. In particular, like justification of a belief that is supported by more and more evidence, the justification for this final step in the explanatory schema is more and more justified the more the minimal conditions are met by the real-world system(s). In the case of statistical modeling, this means that the more events and the more statistically independent those events are, the more justification for the third step of the explanatory schema there will be. Moreover, like justification in other contexts, the justification provided by meeting these conditions to some degree will be difficult to evaluate in borderline cases. However, it is sufficient for us to get a better handle on what the source of justification is in more clear-cut cases. Of course, establishing criteria for the sufficiency of the approximation will be extremely difficult and will certainly depend heavily on the modeling context; e.g. the modeler's explanandum, the nature of the finite real-world system(s), and various disciplinary standards. Nevertheless, we think appealing to the mathematically precise connections between minimal conditions and macroscale behaviors established by paying attention to limit theorems can help clarify what philosophers intend when they appeal to these similarity or approximation relations.[26]

It is also worth mentioning that, similar to the third step in our schema, on Hughes's DDI account, "interpretation is a function that takes us from what we have demonstrated, the necessary consequents of the [models], back into the world of things" (Hughes 1997, p. S333). However, unlike Hughes's account we do not think that this application step is the inverse of the first step of the modeling process in the cases we have described—i.e. a form of deidealization. Instead, appealing to minimal conditions and deriving consequences of those conditions allows us to justify the use of the highly idealized model to develop an explanation without having to de-idealize the model. Moreover, while Hughes does attempt to describe various steps of the modeling process, he says relatively little about what justifies scientific modelers in making each of those steps. As we have argued throughout this paper, we think this is precisely where philosophers of science ought to be focusing more of their attention.

Finally, we want to emphasize that the identification of the above explanatory schema and the conditions involved in justifying the steps of that process is a direct consequence of our alternative philosophical methodology for investigating scientific explanations. We did not arrive at these results by formulating a general account of all instances of explanation, or by demarcating the necessary and sufficient conditions required for being a distinctive kind of scientific explanation. Rather, we arrived at the steps of the schema and the justification for each of those steps by directly analyzing several historically important cases (from a range of sciences) that appear to be explaining similar kinds of explananda in similar ways. Indeed, our approach has yielded these results in virtue of looking directly at the processes used by actual scientific modelers and the implicit justifications appealed to in the distinct contexts of

[26] Thanks to an anonymous reviewer for pressing us to be clearer about this point and the challenges with discussing approximation relations.

the modeling process en route to formulating explanations of real-world phenomena. This illustrates the fruitfulness of our methodology.

4 Accumulating justification for using idealized models to develop explanations

While we have been focusing on the justification for each of the steps of the explanatory schema individually, another unique feature of our account is the way in which idealized modeling is justified more generally. The important thing to note here is that there are two kinds of *cumulative* justification provided by applying the three steps of the schema. First, as each step in the explanatory schema is successfully completed, this provides additional justification for past and future actions involved in completing the other steps of the schema. For example, successfully deriving a result from the mathematical model that is approximated by the explanandum can justify the previous decision to make idealizing assumptions in order to apply that mathematical framework. Moreover, being able to derive that result justifies attempting to provide an explanatory interpretation of those modeling results.

In addition to this cumulative justification within a particular application of the explanatory schema, when the explanatory schema yields an explanation, a boost in justification also transfers to the application of the explanatory schema to similar phenomena in the future. In other words, the justification for using an idealized model to attempt to explain similar explananda in the future can be strengthened or weakened over time by investigating whether the patterns of behavior exhibited by the model are found to hold in additional systems. As more and more instances of those patterns are discovered and explained, that provides evidence that the model system is a member of an interesting (and large) class of systems that display similar patterns of behavior despite differences in their physical components (Batterman and Rice 2014). Consequently, as more instances of the pattern are discovered in real-world systems, there is a justificatory boost provided for the use of the explanatory schema to develop an explanation of similar explananda in the future.

Precisely this kind of reasoning is illustrated by a recent attempt to justify the application of homogenization modeling techniques that have been used to explain phase transitions (and other physical phenomena) to the investigation of the development of arctic melt ponds (Hohenegger et al. 2012; Golden 2014). By discovering that certain minimal conditions are met by these arctic melt ponds, these modelers contend that they are justified in applying various mathematical modeling techniques that have been used to explain other similar phase transition phenomena in physics. As Golden explains, the behavior of these melt ponds:

> …is similar to critical phenomena in statistical physics and composite materials. It is natural, therefore, to ask if the evolution of melt pond geometry exhibits *universal characteristics* that do not necessarily depend on the details of the driving mechanisms…Fundamentally, the melting of Arctic sea ice is a phase transition phenomenon…We thus look for features of melt pond evolution that are mathematically analogous to related phenomena in the theories of phase transitions and composite materials. (Golden 2014, p. 13)

What we see here is the attempt to justify an application of a previously successful explanatory schema—in this case the application of homogenization techniques—to a new phenomenon because of similarities between the explananda and the minimal conditions met by the systems. In other words, these modelers claim to be justified in applying this idealized modeling framework because the schema has been successfully used to develop explanations of similar kinds of phenomena in the past and they have discovered that the same minimal conditions are met by the current system of interest. As a result, the kind of inductive inference that has been used to justify the reliability of statistical modeling techniques can also be used to accumulate inductive justification for future applications of other (non-statistical) explanatory schemas across scientific disciplines.

5 Distinguishing our approach from other methodologies for investigating explanation

In this section, we contrast our account with other extant methodologies for investigating the explanations provided by scientific models. In particular, we contend that philosophers' attempts to provide general accounts of explanation, or to demarcate kinds of explanation, has led to overemphasis on whether or not the explanation provided by an idealized model fits a particular account (of a kind) of explanation; e.g. whether the explanation provided is causal or statistical. We will argue that this overemphasis has led philosophical analyses of instances of explanation to miss significant features of the modeling process that justify the use of an idealized model to formulate an explanation; e.g. the role played by minimal conditions, limit theorems, and approximation in statistical modeling.

5.1 The Hempelian project

In the philosophy of science, serious inquiry into the nature of scientific explanation began with Hempel (1965). Hempel wanted to give necessary and sufficient conditions for when something counted as a scientific explanation. Because Hempel tried to offer necessary and sufficient conditions for explanations, objections to his view came in the form of numerous counterexamples.[27] Regardless of its shortcomings, Hempel's general project of providing necessary and sufficient conditions set the standard for how to investigate the nature of scientific explanation. Indeed, Hempel's methodological approach was later adopted by most philosophers of science who attempted to analyze the nature of scientific explanation (e.g. see Craver 2006; Friedman 1974; Kitcher 1981; Railton 1981; Salmon 1984; Strevens 2009; Reutlinger 2016).[28]

27 For example, Hempel's account allows for shadows, together with some laws of optics, to explain the height of flagpoles (Bromberger 1966).

28 While there may be multiple ways to satisfy those criteria—e.g. Hempel discussed both deductive nomological and inductive statistical explanations—the Hempelian project is, essentially, to attempt to identify the necessary and sufficient conditions that distinguish explanations from nonexplanations.

A contemporary example of this approach to scientific explanation is Michael Strevens's kairetic account (Strevens 2004, 2009). Accordingly, scientific explanations are causal models that include all and only the difference-making causal factors (Strevens 2009, pp. 73–75, 123). Like many others, Strevens's focus on causation is motivated by various counterexamples to Hempel's DN-model. He says: "The obvious candidate" for solving the flagpole problem "is the relation of causation" (Strevens 2009, p. 25). However, Strevens argues that only certain causes are explanatorily relevant. That is, according to the kairetic account, "the explainers of a phenomenon must both bear the causal relation to the phenomenon and pass a test for explanatory relevance" (Strevens 2009, p. 21). The explanatorily relevant causes are picked out by Strevens's kairetic criterion, which focuses on isolating the causal factors that made a difference to the occurrence of the explanandum. Consequently, according to Strevens's account, "A standalone explanation of an event *e* is a causal model for *e* containing only difference-makers for *e*" in which, "the derivation of *e*, mirrors a part of the causal process by which *e* was produced" (Strevens 2009, pp. 73–75). Strevens then uses his account of the nature of scientific explanation to interpret how idealized models provide causal explanations by accurately representing difference makers and only distorting irrelevant causes.

While we think there are many strengths to Strevens's account of causal explanation, we think his account of the explanatory product leads to normative claims about scientific modeling that mischaracterize the justification of the techniques used by modelers in many cases. In particular, the way Strevens thinks about explanation influences his account of what justifies the use of idealized models to develop explanations. Specifically, on Strevens's account, idealizations are justified in models that explain when they distort irrelevant causes in order to isolate an accurate representation of the causal difference makers:

> The content of an idealized model, then, can be divided into two parts. The first part contains the difference-makers for the explanatory target...The second part is all idealization; its overt claims are false but its role is to point to parts of the actual world that do not make a difference to the explanatory target. The overlap between an idealized model and reality...is a standalone set of difference-makers for the target. (Strevens 2009, p. 318)

Strevens also adds, "All idealizations, I suggest, work in the same way" (Strevens 2009, p. 341). However, in the cases above, Galton, Maxwell and Fisher are not using idealizations to emphasize the irrelevance of the distorted features so as to isolate the causal difference makers. Instead, these modelers introduce these idealizations in order to allow for the use of the mathematical modeling techniques available—even when the idealizations distort many causal factors that *do* make a difference to the explanandum. For example, Fisher's statistical models distort the difference-making processes of natural selection (and drift) by representing them as occurring in an infinite population where mating is completely random. This statistical modeling technique is a key part of what Fisher borrowed from Maxwell's modeling of gas behavior. Similarly, Galton distorts the causal difference makers in the population by assuming that their interactions are completely independent of one another. Furthermore, rather than isolating causal difference makers from other irrelevant causes, the modeling techniques involved in these cases aggregate over all the causes (or causal trajectories) of the population in order to focus on the stable statistical properties of the overall population. Indeed, any causal difference makers at the level of individual genes, particles, or people become irrelevant once we move to a statistical modeling framework involving (effectively) infinite populations.[29] As a result, universally applying Strevens's account of what makes for a good explanation would lead to the mischaracterization of the justifications that ought to be provided for many of the statistical modeling techniques used to formulate explanations in science. These modelers are justified in making various idealizing assumptions because they enable them to use certain mathematical modeling techniques that are particularly fruitful for investigating the kind of explananda they are interested in explaining.

While Strevens might argue that, in the Fisher case, numerous genes and random mating are the causal difference makers, these minimal conditions that justify the use of the idealized model are not accurately represented in the way required by Strevens's account. As we noted above, these features are only approximated to a certain degree by the finite real-world system in such a way that the system will also approximate various macroscale patterns of behavior exhibited by the idealized model. Furthermore, while the macroscale pattern does counterfactually depend on these statistical features of the population, counterfactual dependence does not necessarily entail causal dependence.

5.2 Demarcating kinds of scientific explanation

Unlike Strevens, Lange (2012, 2013) does not try to provide a general account of explanation; rather, he attempts to provide necessary and sufficient conditions for distinctive *kinds* of explanation.[30] Instead of replacing Hempel's view with an alternative account of scientific explanation (e.g. in terms of causation), this methodology attempts to analyze several different kinds of explanation by identifying what distinguishes each kind of explanation from the others. For example, in a recent paper, Lange argues that there is a distinctive kind of explanation that appeals to statistical facts—he calls these explanations "really statistical" (RS) explanations.[31] Lange writes: "I argue that really statistical (RS) explanation is a hitherto neglected form of scientific explanation" (Lange 2013, p. 169).[32] After analyzing the difference between causal and really statistical explanations, Lange concludes, "an explanation is RS if and only if it works by identifying the result being explained as an instance of some characteristically statistical phenomenon such as regression toward the mean" (Lange 2013, pp. 177–178). Here we see that Lange's goal is to offer a set of necessary and sufficient conditions for the kind of explanation he calls "really statistical".

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29 Thanks to an anonymous reviewer for helping us clarify this point.

30 Pincock (2007, 2012) adopts a similar methodology by attempting to demarcate mathematical explanations from causal explanations.

31 In addition, Lange argues that there are distinctively mathematical explanations (Lange 2012).

32 We find this claim puzzling given that statistical explanations have been discussed in the philosophy of biology literature (Hacking 1990; Matthen and Ariew 2002, 2009; Sober 1980; Walsh et al. 2002).

Unfortunately, if we try to use Lange's account of statistical explanation to characterize the cases above, it either mischaracterizes or misses the features that actually justify the modeling processes involved in using a statistical model to formulate an explanation. On one interpretation, Lange's account might be used to justify the use of a statistical model to explain by arguing that the real-world system *instantiates* a particular statistical result that is also instantiated by the statistical model. Perhaps this is what Lange means by showing that the phenomenon is "an instance of some characteristically statistical phenomenon". If we go with this interpretation, using Lange's account in this way would seem to mischaracterize the justification of the modeling process in the cases we presented above because the exact statistical result that is derived within the idealized mathematical model is only *approximated* to various degrees by the real-world system(s). That is, applying Lange's account to the cases of Galton, Maxwell, and Fisher would mistakenly suggest that the justification for using these statistical models to explain is that the phenomenon of interest instantiates the same statistical results as the idealized model system, but this perfect mirroring of the statistical process almost never holds within idealized statistical modeling.

Given his focus on determining whether a resulting explanation is "really statistical", it isn't surprising that applying Lange's account to these cases seems to provide little (if any) information about what justifies using a statistical model to formulate an explanation. On its own this isn't a serious objection given Lange's goals, but it is important to see that trying to use Lange's account of the explanatory product to analyze these three cases would lead to the mischaracterization of the structure of how these explanations work. Specifically, Lange's account would fail to recognize the essential role played by minimal conditions and limit theorems that guarantee (to the degree they are met) that the real-world system will approximate the statistical results derived in the idealized case. In other words, it isn't just that Lange's account fails to answer these alternative questions about justification and the process of developing an explanation, but that by not addressing them, his account misses essential features of the resulting explanations provided in these cases. For example, in Galton's case, Lange's account would miss the role of the central limit theorem in guaranteeing that systems that meet certain minimal conditions will approximate the normal curve, but it is precisely these features that are crucial to understanding Galton's explanation of the stability of the bell curve across generations. In addition, the central limit theorem is essential to showing that the statistical modeling techniques used by Fisher and Maxwell will be approximated by any system that meets certain minimal conditions.

In short, Lange's attempt to determine when a resulting explanation counts as "really statistical" either mischaracterizes or misses the role of minimal conditions, limit theorems, and approximation in using these statistical models to explain. The more general issue is that by focusing on providing a general account of what all statistical explanations have in common, all Lange's account provides is a rather vague requirement of showing that the phenomenon is an "instance of some characteristically statistical phenomenon". This is rather unhelpful in fleshing out the details of particular cases of statistical explanation in science. Only by looking at the details of particular cases and extracting the steps and justifications involved in their modeling process en route to developing their explanations, can we hope to provide a more detailed (and accurate) description of the explanatory structure of these cases.

6 Conclusion

In this paper we have argued that rather than exclusively focusing on trying to determine if the explanation provided by an idealized model fits a general account of scientific explanation, or is a particular kind of scientific explanation, philosophers of science should focus more attention on directly investigating the steps and justification involved in various explanatory schemas used to develop explanations by appealing to highly idealized models. Throughout this paper, our focus has been on investigating three questions: (1) What are the steps used by scientific modelers who cite idealized models in their explanations? (2) What justifies each step of their modeling process? (3) What conditions are involved in allowing for the modeling process to provide results that are serviceable for explanation in these (and other future) cases? We think that directly investigating these questions will allow for an improved understanding of when and why idealized models can be used to develop scientific explanations and avoid mischaracterizing historically important exemplars of scientific explanation. As a result, we hope that the adoption of our alternative methodology will breathe new life into debates concerning scientific explanation and we encourage philosophers of science to use this methodology to extract additional explanatory schemas in the future.

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